AIAA 80-1644R

Earth's Magnetic Field Models for Dumping Momentum Magnetically on GPS Satellites

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This paper discusses the relative performance of various Earth magnetic field models in a system for using on-board electromagnets to dump unwanted reaction wheel momentum on Global Positioning System satellites. The system uses the interaction between the Earth's magnetic field and the satellite's electromagnetically induced field to control the reaction wheel angular momentum vector using magnet switch-time and power settings produced by a ground-based computer program. The control performance and computational efficiencies of four magnetic field models are compared: the Mead-Fairfield (used in the original program), a quadrupole, a tilted dipole, and a spin dipole model. Results indicate that savings in computer time can be obtained using the simpler magnetic field models with very little loss of performance. Data have been verified by testing on actual satellites in orbit.

Introduction

THE Global Positioning System (GPS) is a planned worldwide navigation system which provides users with three-dimensional location information accurate to within 10 m. The space segment of this system will consist of 18-24 three-axis stable, Earth-pointing satellites in 12 h circular orbits inclined at 55 deg where the distance from the center of the Earth to the satellite is 4.17 Earth radii. To date, six satellites have been launched into 63 deg orbits as part of the full-scale engineering development of the program. The magnetic momentum dumping system currently in use was tested extensively during the fall of 1979 on the four satellites in orbit at that time. These tests demonstrated that momentum built up over a period of two weeks could be dumped magnetically by choosing nine magnet switch times over a two-revolution cycle and setting the magnets at each switch time to a predetermined optimal value between ±7500 pole-cm. Since that time, the system has been demonstrated effective using five magnet switch times over a one-revolution cycle with magnet settings between ±15000 pole-cm. Magnetic momentum dumping is presently being used on all six GPS satellites as the primary method of reducing unwanted momentum.

Momentum dumping is required on GPS satellites because external torques build up momentum on the on-board reaction wheels. When these wheels reach certain limits, a system is needed to dump (reduce) the wheel momentum. Originally, the planned system for dumping momentum on these satellites consisted of hydrazine control jets. The current magnetic dumping system uses the interaction between the satellite's on-board electromagnets and the Earth's magnetic field to gradually dump reaction wheel momentum (over 1-2 revolutions). The use of the magnetic momentum dumping system instead of the hydrazine control jet system has two primary advantages for the GPS. First, stored hydrazine carried up from the ground is no longer depleted through the use of the control jets for momentum dumping. Second, because the control jet system thrusters are not used, there is no net Delta-V imparted to the satellite, so the satellite ephemeris is not disturbed at all by dumping momentum. Previously, the use of the hydrazine control jets introduced ephemeris errors that made the navigation signal unusable for up to 24 h once every two weeks.

In 1974, when this magnetic momentum dumping system was first proposed for use on GPS it was widely believed that the Earth's magnetic field was too uncertain and variable at the altitude of the GPS satellites to be used in an open-loop (i.e., without actual measurements of the magnetic field at the satellite) mode for computing magnet settings and switch times. The Earth's magnetic field vector in the vicinity of synchronous equatorial satellites was said to vary at times up

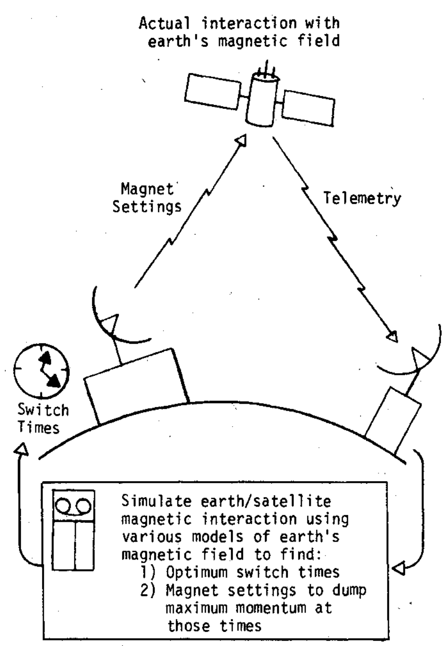

Fig. 1 Testing of various Earth magnetic field models for GPS magnetic momentum dumping.

Presented as Paper 80-1644 at the AIAA/AAS Astrodynamics Conference, Danvers, Mass., Aug. 11-13, 1980; submitted Aug. 11, 1980; revision received April 30, 1982. This paper is declared a work of the U.S. Government and therefore is in the public domain.

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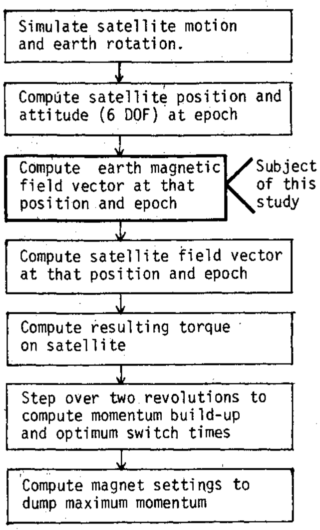

Fig. 2 Use of Earth magnetic field model in magnetic momentum dumping algorithm.

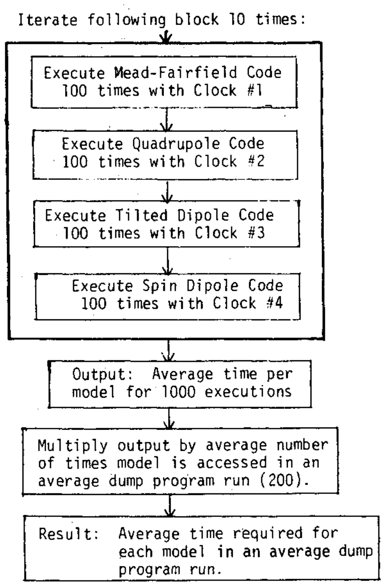

Fig. 3 Determining computation time required by magnetic models in momentum dumping program.

to 40% in magnitude and 180 deg in direction, particularly during increased solar activity.[1] In order to simulate the magnetic environment as realistically as possible, a Mead-Fairfield model was used in the original program because it was based on magnetometer data and accounted for the direction and activity level of the sun.[2] Although the magnetometer data were taken primarily beyond 5 Earth radii—well above the altitude of the GPS satellites—the Mead-Fairfield model was found to work very well during numerous tests on GPS satellites during the fall of 1979.[3]

After it was demonstrated that the magnetic mometum dumping algorithm worked well with on-orbit GPS satellites using the Mead-Fairfield magnetic model, this study was performed to determine the comparative efficiency and performance of simpler mathematical models for the Earth's magnetic field. A general illustration of the entire magnetic momentum dumping system is portrayed in Fig. 1. A flow chart describing where this study fits into the overall magnetic momentum dumping algorithm is shown in Fig. 2. Since no measurement of the Earth's magnetic field is made at the spacecraft, this quantity must be inferred from a mathematical model which uses spacecraft position relative to a rotating Earth as an input. The Mead-Fairfield model also uses the location of the sun, whereas the other models tested in this study needed only satellite position. The momentum vector to be dumped was based on telemetry data of the satellite's actual momentum state.

Table 1 Average difference between optimum switch times determined by three magnetic models and the Mead-Fairfield model, min

Model	*X* axis	*Y* axis
Quadrupole	1.17	1.45
Tilted dipole	0.35	0.41
Spin dipole	8.67	10.24

This study was undertaken with the assumption that the Mead-Fairfield model, having been successfully tested, was a relatively suitable model of the actual Earth's magnetic field (at GPS satellite altitude). With that in mind, this study attempted to answer the following questions:

1) Is there a simpler model of the Earth's magnetic field that will give performance similar to the Mead-Fairfield model?

2) How much computational time can this simpler model save?

Magnetic Models

In addition to the Mead-Fairfield model, three simple magnetic field models were studied: a quadrupole model, a tilted dipole model, and a spin dipole model. These models are explained below.

1) Mead-Fairfield model. Derived from the least squares fit to magnetic field measurements from 451 orbits of four satellites. Measurements were taken between 4 and 17 Earth radii. This model is described by a power series expansion and requires sun position as an input.[4]

2) Quadrupole model. Assumes that the Earth's field is made up of two magnets, one north-south and one in the equatorial plane. This model is described by the first eight terms of a spherical harmonic expansion.[5]

3) Tilted dipole model. Assumes that the Earth's field is a dipole tilted away from the north-south spin axis. This model is described by the first three terms of a spherical harmonic expansion.[5]

4) Spin dipole model. Assumes that the Earth's field is a simple dipole aligned along the north-south spin axis. This model is described by the first term of a spherical harmonic expansion.[5]

Testing

These models were tested as follows. First, the computer program for finding optimum switch times was run for test cases of past actual dumps using each magnetic field model. Comparisions with the Mead-Fairfield model are indicated in Table 1. The average time differences between models vary, but the largest average difference, approximately 10 min for the spin dipole model, is still quite small when considering that the switch times are 3-4 h apart and are constrained by competing requirements for transmitter site scheduling. Also, the step size used in the time program is 4 min, so average differences less than this are not very meaningful. For these reasons, it was felt that the difference in optimum switch times between models would not have a significant effect on the amount of momentum dumped. The switch times determined by the Mead-Fairfield model (as filtered through site scheduling constraints) were, therefore, used for all magnet setting computations in later testing.

The magnet setting computations of the three models were then tested for 21 momentum dumping cases. For each test, the epoch satellite position and momentum data were entered into the program, along with the scheduled switch times and the Earth magnetic model being used. The program then computed the optimum magnet settings to dump the most momentum for the specified Earth model. The program then took these settings and, using the Mead-Fairfield model as the best "real-world" estimate, predicted a time history of momentum at each scheduled switch time. This predicted time history provided the data used in this study to compare the performance of the different magnetic field models to the Mead-Fairfield model and to the actual telemetry history of momentum from the satellites.

This study also looked at the relative computational efficiency of the various models. The subroutines required for each model were executed 1000 times using elapsed time clocks as indicated in Fig. 3. This approach was used to minimize timing errors due to load mix on the computer and discretization error in the timing clocks. The average time required per model for actual momentum dump computations was then multiplied by the average number of times the model has to be accessed in actual momentum dump program runs (approximately 200). These calculations produced the amount of computer time, on average, that each model would require if used for an average full program run.

Comparison of Model Predictions

The optimum switch times determined from each model were compared to the Mead-Fairfield model (see Table 1). Although these differences are small and were not included in our magnet setting computations, Table 1 does show a trend. The model that most closely compares to the Mead-Fairfield in optimum switch times was the tilted dipole model. The next closest was the quadrupole model, and the least comparable was the spin dipole model.

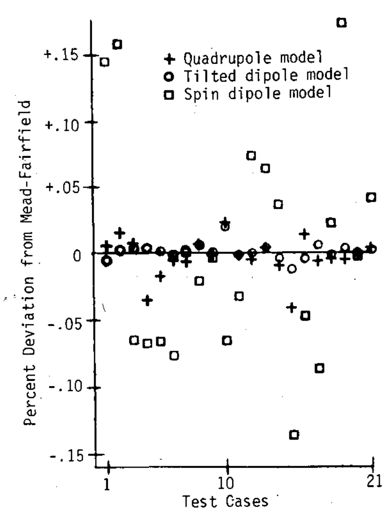

Fig. 4 Predicted momentum dump compared to the Mead-Fairfield model.

The results of 21 momentum dumping cases are compared in Table 2 and Fig. 4. In this figure, the Mead-Fairfield model is used as the nominal or reference prediction. This figure displays graphically how well the simpler magnetic field models compare to Mead-Fairfield. The tilted spin dipole model compares very closely to the Mead-Fairfield prediction, with an average deviation of ±0.4%. The quadrupole model is the next closest, with an average deviation of ±1.0%. Finally, the spin dipole model least favorably compares, having very large (±15%) deviations from the Mead-Fairfield reference. These results were not entirely expected. One would expect the quadrupole model to be a more accurate predictor than the tilted dipole model, since it is a more complex model. The tilted dipole, however, seemed to compare more closely to the Mead-Fairfield predictions than did either the quadrupole or spin dipole model.

The results of the test on the amount of computation time required for the various models were generally as expected. The amount of time required decreased as the complexity of the model decreased. The quadrupole model is four times faster than the Mead-Fairfield, the tilted dipole is five times faster, and the spin dipole is six times faster. See Table 3 for a compilation of the results.

Actual On-Orbit Results

Generally, due to limited transmitter site availability, use of less-than-optimum switch times, magnet strength limits, etc., most predictions are for less than 100% momentum dump. On the average, 60-90% of the momentum is predicted to be dumped. Of that prediction, normally 60-100% is actually dumped. The reasons why the amount of actual momentum

Table 2 Deviations of model predictions from Mead-Fairfield model, % of momentum dump predicted

Model	Average deviation from Mead-Fairfield	Maximum deviation from Mead-Fairfield
Quadrupole	1.0	4.0
Tilted dipole	0.4	2.0
Spin dipole	6.6	17.3

Table 3 Processor time required for average momentum dump program run (200 calls on subroutine)

Model	Average time required, s	Reduction over Mead-Fairfield, %
Mead-Fairfield	4.97	–
Quadrupole	1.34	73
Tilted dipole	1.02	79
Spin dipole	0.84	83

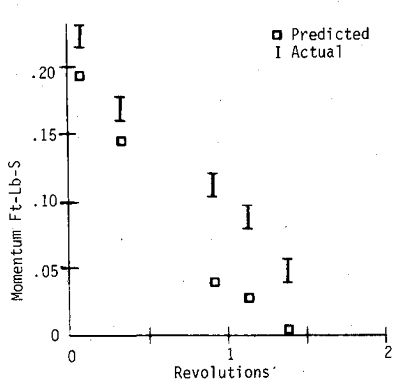

Fig. 5 Predicted and actual momentum using the quadrupole model, satellite 5112, May 25, 1980.

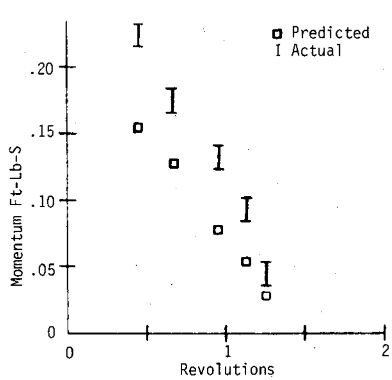

Fig. 6 Predicted and actual momentum using the tilted dipole model, satellite 5117, May 22, 1980.

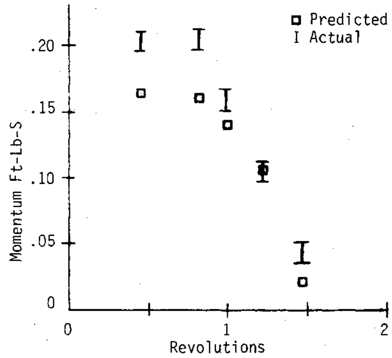

Fig. 7 Predicted and actual momentum using the spin dipole model, satellite 5113, May 21, 1980.

dumped does not more closely follow the predictions are many and range from simplified modeling of the system (no momentum growth rate is modeled between momentum data epochs and the actual switch times, for instance) to noise in the site-satellite-telemetry loop. While not addressed here, future research in these areas could be very productive.

The magnetic model testing was verified by using the magnet setting commands from each model on several on-orbit satellites. Generally, within the rather wide limits discussed above, the results of different magnetic models agreed with the amount of momentum dump predicted. Figures 5-7 show three of the test cases. The graphs display actual vs predicted momentum change for quadrupole, tilted dipole, and spin dipole models. Each vertical data pair occurs at a switch time, with the square symbol denoting the predicted total momentum at that switch time, and the I symbol denoting the actual total momentum at that time. The height of the I symbol represents the width of the least significant bit in the telemetry bit stream.

Conclusion

The tilted dipole model appears to work about as well as the Mead-Fairfield model at the GPS satellite altitude. Optimum switch times are within 1 min and predicted momentum dumped is within 0.4%. Compared to the Mead-Fairfield model, the tilted dipole used 80% less computer time for calculating the Earth's magnetic field. Thus, the tilted dipole model could provide improved computational efficiency in GPS magnetic momentum dumping calculations with very little loss in performance. This could ultimately lead to the capability for on-board computations in small microcomputers.

Acknowledgments

The authors acknowledge the ongoing assistance of the U.S. Department of Defense Global Positioning System Joint Program Office and the work on magnetic momentum dumping by faculty and cadets at the Air Force Academy between 1974 and 1980.

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